Hopf Algebras and Related Topics*

a mini-conference held at

Auburn University at Montgomery
Montgomery, Alabama, USA

March 13-14, 2019

* The organizers thank the Department of Mathematics and Computer Science at Auburn University at Montgomery for its support.

Schedule

Wednesday, March 13, Goodwyn Hall 201

9:45am–9:55am	*Opening remarks*
10:00am–11:00am	*Bi-skew braces and Hopf Galois structures* Lindsay N. Childs, University at Albany, SUNY
1:30pm–2:30pm	*Kummer Theory for models of diagonalizable group schemes* Dajano Tossici, University of Bordeaux
3:00pm–4:00pm	*Hopf orders in* $(KC_p^3)^*$ *over a discrete valuation ring of characteristic* p Robert Underwood, AUM

Thursday, March 14, Goodwyn Hall 204

9:00am–10:00am	*Quotient Hopf-Galois structures and associated orders in Hopf-Galois extensions* Paul J. Truman, Keele University
10:30am–11:30am	*Subextension techniques to describe Hopf Galois structures* Ana Rio, Barcelona School of Informatics
1:00pm–3:00pm	*Informal work session*

Abstracts

Bi-skew Braces and Hopf Galois Structures

Lindsay N. Childs

A skew left brace is a finite set $G = (G, \star, \circ)$ with two group operations that satisfy a compatibility condition somewhat analogous to distributivity. Skew left braces were introduced by Guarneri and Vendramin (2017) and Bachiller (2016), generalizing Rump's (2007) concept of a left brace (in which the "additive group" $(G, \star)$ is abelian). Extending an observation of Bachiller on left braces, Byott and Vendramin showed that given a pair of finite groups (Γ, N) of equal order, there is a Hopf Galois structure of type N on a Γ-Galois extension of fields if and only if there is a skew brace $(G, \star, \circ)$ with additive group $(G, \star) \cong N$ and circle group $(G, \circ) \cong \Gamma$. Thus the existence problem: which ordered pairs (Γ, N) can arise in a skew brace, is of interest to both skew brace theorists and Hopf Galois theorists.

This talk will review this work and then introduce bi-skew braces, skew braces in which either group can play the role of the additive group. Much of the talk will explore examples.

Kummer Theory for models of diagonalizable group schemes

Dajano Tossici

In this talk I will speak about a joint work in progress with Matthieu Romagny. It is well known that any diagonalizable group schemes is endowed with a Kummer Theory which let us to describe easily torsors under these group scheme. Let R be a discrete valuation ring with residue field of positive characteristic p. In this talk I explain how to construct group schemes over R with diagonalizable generic fiber, in a such way to extend the Kummer theory to the whole group scheme. I will exhibit a large family of such models which, conjecturally, includes any model of a diagonalizable group scheme.

Hopf orders in $(KC_p^3)^*$ over a discrete valuation ring of characteristic p

Robert Underwood

Let p be prime, let R be a discrete valuation ring of characteristic p and quotient field K. Let C_p^n denote the elementary abelian group of order p^n. Let KC_p^n be the group ring Hopf algebra with dual Hopf algebra $(KC_p^n)^*$. This talks concerns the structure of R-Hopf orders in $(KC_p^n)^*$ for $n \geq 1$. The cases $n = 1, 2$ are known; complete classifications have been given by J. Tate and F. Oort in the case $n = 1$, and G. Elder and U. in the case $n = 2$. For $n = 1$, one parameter is required to determine the Hopf order, and for $n = 2$ we require three parameters. For arbitrary n, A. Koch has recently shown that Hopf orders in $(KC_p^n)^*$ are completely classified using $n(n+1)/2$ parameters. What remains unsettled is the explicit structure of the Hopf orders in $(KC_p^n)^*$ (and their duals in KC_p^n). Towards this end, we determine the algebraic structure of all Hopf orders in $(KC_p^3)^*$ and conjecture about the structure of their duals in KC_p^3.

Quotient Hopf-Galois Structures and Associated Orders in Hopf-Galois Extensions

Paul J. Truman

Let L/K be a finite Galois extension of fields with group G. By the Greither-Pareigis classification, each Hopf algebra giving a Hopf-Galois structure on L/K has the form $L[N]^G$ for some regular subgroup N of $\mathrm{Perm}(G)$ that is normalized by $\lambda(G)$, the image of G under the left regular representation. If M is a subgroup of N that is also normalized by $\lambda(G)$ then $L[M]^G$ is a Hopf subalgebra of $L[N]^G$, which gives rise to a fixed field L^M and a Hopf-Galois structure on L/L^M. In recent work with Koch, Kohl, and Underwood, we showed that if in addition M is a normal subgroup of N then M gives rise to a "quotient" Hopf-Galois structure on L^M/K. In this talk we study some applications of this construction to questions of integral Hopf-Galois module structure in extensions of local or global fields.

Subextension techniques to describe Hopf Galois structures

Ana Rio

We study the Hopf Galois property in subfields of a given field extension. For a finite Galois extension having a Galois group which is a semidirect product, we are able to construct split Hopf Galois structures induced from suitable subfields. We see how all the split structures are obtained by this induction process. On the other hand, we will consider some examples on how induction can also be combined with the theorem of Koch, Kohl, Truman and Underwood on normality to obtain the whole picture of Hopf Galois types. This is joint work with T. Crespo and M. Vela