Math 372 Exam 4 Total pages: 5 Total points: 70
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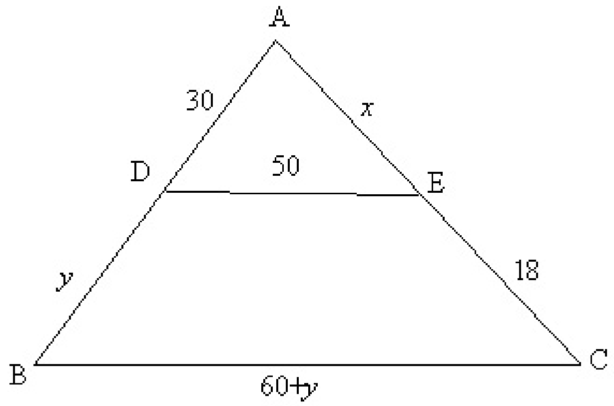
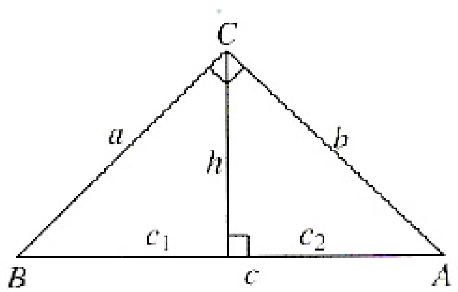
Name(Print)________________ Section ________________ Grade___________
Attention: **Answers without supporting work shown on the paper will receive NO credits.**

1. In $\triangle ABC$, $\overleftrightarrow{DE} \| \overleftrightarrow{BC}$ and certain measurements are as indicated. Find x and y.



2. Given a right triangle ΔABC, the lengths of the two legs are a and b respectively and the length of the hypotenuse is c. Draw an altitude from the vertex C to the hypotenuse. The two segments on the hypotenuse cut off by the altitude are of lengths c_1 and c_2 respectively with c_1 close to vertex B.



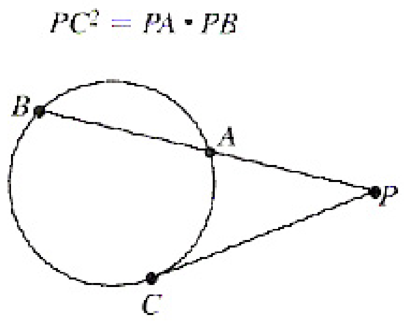
Show

$$h^2 = c_1 c_2.$$

3. Prove the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices of the right triangle using the **formula for the length of a median-the Cevian Formula**.

4. Prove the Secant-Tangent Theorem. If a secant $\overleftrightarrow{PA}$ and tangent $\overleftrightarrow{PC}$ meet a circle at the respective points A, B, and C (point of contact), then in the following figure,

$$PC^2 = PA \cdot PB$$



5. **The Power of a Point** The **power** of a point P with respect to a circle with center O and radius r is the real number

$$\text{Power}(P) = PO^2 - r^2$$

(a) Prove that if P lies outside circle O and $\overleftrightarrow{PT}$ is tangent to the circle at T, then

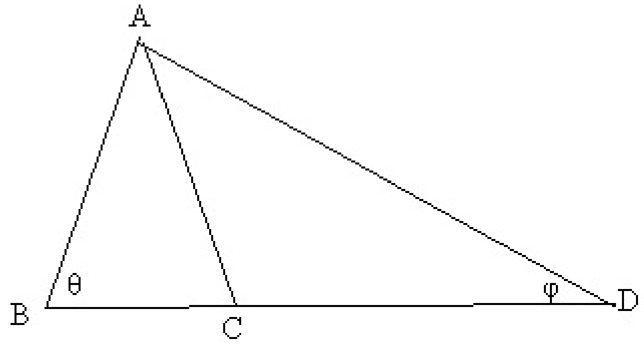
$$\text{Power}(P) = PT^2.$$

(b) Identify the set of all points P for which

$$\text{Power}(P) = k,$$

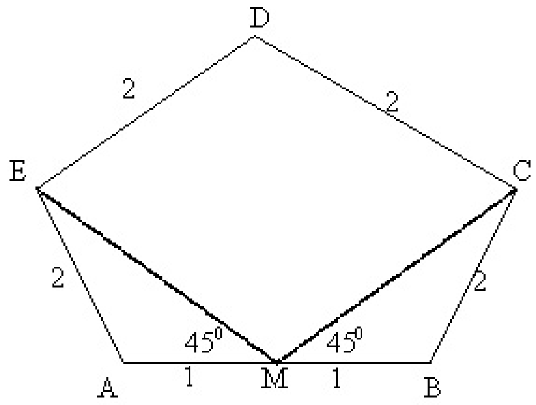
where k is a constant.

6. **Golden Isosceles Triangle** Isosceles triangle ΔABC lies within another isosceles triangle ΔABD ($AB = AC$ and $AD = BD$). In addition, it is given that ΔACD is isosceles with $AC = CD$



(a) Show that this can happen (and will happen) precisely when ΔABC is a **Golden Isosceles Triangle**, that is, when $AB = \tau BC$, where $\tau = \frac{1}{2}(1+\sqrt{5})$-the Golden Ration. (Hint: Use similar triangles to show that $AB^2 = BC \cdot BD = BC^2 + BC \cdot AB$, which can be converted into the equation $x^2 = x + 1$ where $x = AB/BC$.
(b) Show that $\theta = 72$ and $\phi = 36$ in the figure.
(c) Using trigonometry in ΔABC, show that $\sec 72^o = 2\tau$ and $\cos 36^o = \tau/2$.

7. **The Geometry of the Cario Tile** The following is a picture of so called **Cario tile** with measures as indicated.



Show from the laws of Sines and Cosines that $MD = \sqrt{7}$, $MC = (\sqrt{7}+1)/\sqrt{2}$, and $EC = \sqrt{7}+1$. It then follows that $m\angle E = m\angle C = 90^0$ and $m\angle ABC \approx 114^0$. Find $m\angle EDC$ using these estimations.