Math 372 Exam 3 Total pages: 3 Total points: 40
Instructor: Yi Wang

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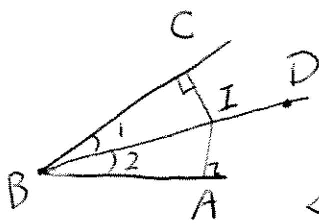
Name(Print)_______________ Section ______________ Grade__________

Attention: **Answers without supporting work shown on the paper will receive NO credits.**

Instruction: Please answer Problems 1-5 within the context of absolute geometry. Problem 6-8 can be answered within the Euclidean geometry.

1. Prove that if I is any point on the bisector $\overline{BD}$ of $\angle ABC$, then I is equidistant from its sides, and conversely.



$\Rightarrow$) Let I be a point on $\overline{BD}$, and $IC \perp BC$, $IA \perp BA$

Need to prove $IC = IA$.

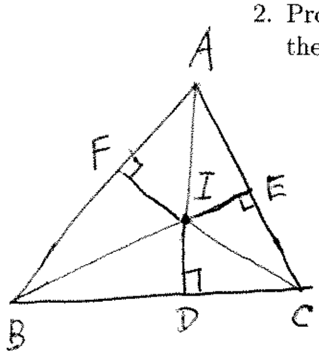
$\left.\begin{array}{l}\angle 1 \cong \angle 2 \\ BI \cong BI\end{array}\right\} \Rightarrow \triangle CBI \cong \triangle ABI \Rightarrow IC = IA$

$\Leftarrow$) Conversely, Let $IC = IA$, $IC \perp BC$, $IA \perp BA$,

need to prove $\angle 1 \cong \angle 2$

$\triangle CBI \cong \triangle ABI$ by HL, so $\angle 1 \cong \angle 2$. ▨

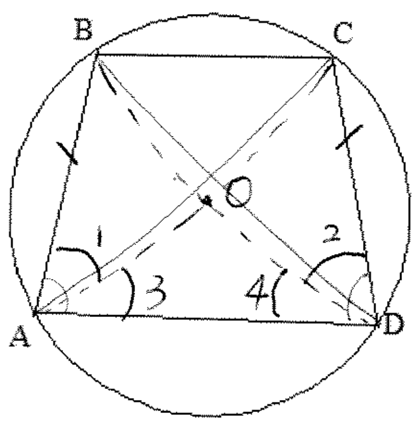
2. Prove that the angle bisectors of any triangle are concurrent at a point I, called the **incenter**, that is equidistant from the three sides of the triangle.



Let $\triangle ABC$ be given, ~~and~~ IA is the angle bisector of $\angle BAC$, ~~and~~ IB is the angle bisector of $\angle ABC$, and ~~as~~ they intersects at the point I. Let $ID \perp BC$, $IF \perp AB$, $IE \perp AC$.

$\left.\begin{array}{l}IF = ID \text{ since } BI \text{ is the angle bisector} \\ IF = IE \text{ since } AI \text{ is the angle bisector}\end{array}\right\} \Rightarrow$ Together they give $IE = ID \Rightarrow$ which shows IC is the angle bisector of $\angle BCA$ by prob. 1 ▨

3. A circle passes through the vertices of $\diamond ABCD$, and $AB = CD$. Prove that $m\angle A = m\angle D$.



Let O be the center of the circle.

$\triangle OAB \cong \triangle ODC$ by SSS

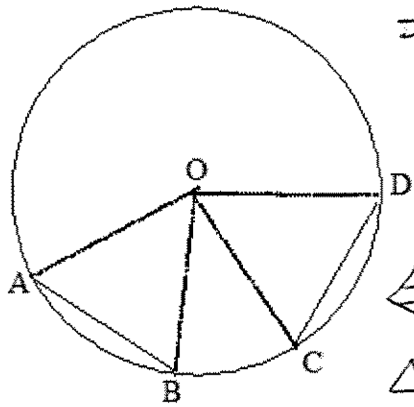
$\angle 1 \cong \angle 2$ by CPCF

$\triangle OAD$ is isoceles triangle

$\Rightarrow \angle 3 \cong \angle 4$

$\Rightarrow \angle BAD \cong \angle CDA$. ▨

4. Prove two chords of a circle are congruent iff they subtend arcs of equal measure. (You must establish $m\widehat{AB} = m\widehat{CD}$ iff $AB = CD$. Recall that $m\widehat{AB} = m\angle AOB$.)



$\Rightarrow$) Assume $m\widehat{AB} = m\widehat{CD}$.

$\Rightarrow \angle AOB \cong \angle COD$

$\Rightarrow \triangle ABO \cong \triangle CDO$ by SAS

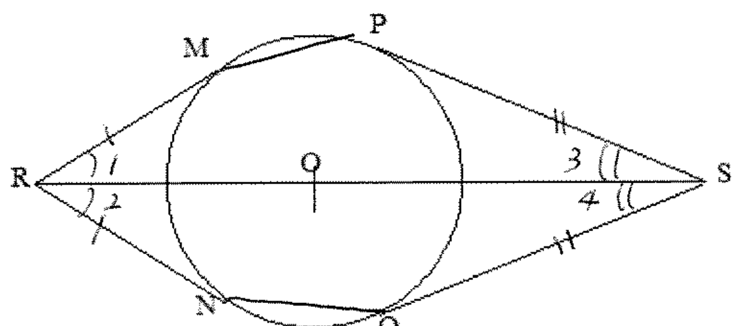
$\Rightarrow AB = CD$ by CPCF

$\Leftarrow$) conversely, assume $AB = CD$

$\triangle ABO \cong \triangle CDO$ by SSS

$\Rightarrow \angle AOB \cong \angle COD \Rightarrow m\widehat{AB} = m\widehat{CD}$ ▨

5. Tangents are drawn to circle O form points R and S, which lie on a line passing through the center O. If M, N, P, and Q are the points of contact, prove $m\widehat{MP} = m\widehat{NQ}$.

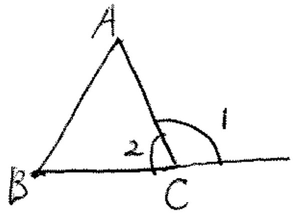


proof It suffices to show $MP = NQ$ by prob. 4.

Note $RM = RN$, $\angle 1 \cong \angle 2$,
$SP = SQ$, $\angle 3 \cong \angle 4$
$RS = RS$
$\Rightarrow \Diamond RSPM \cong RSQN$ by SASAS.

$\Rightarrow MP = NQ$ by CPCF. ▨

6. Prove that In Euclidean geometry, the sum of the measures of the angles of any triangle is 180.



Proof: Let $\triangle ABC$ be given.

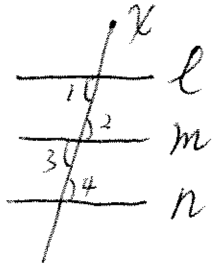
$m\angle 1 = m\angle A + m\angle B$

$m\angle 1 + m\angle 2 = 180$

$\Rightarrow m\angle A + m\angle B + m\angle 2 = 180$

$\boxed{}$

7. **Transitivity of Parallelism in Euclidean Geometry** Prove that for three distinct lines ℓ, m and n, if $\ell \| m$ and $m \| n$, then $\ell \| n$.

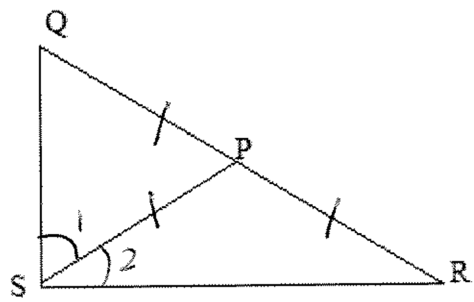


let x be a line transverses ℓ, m, and n.

$$\left.\begin{array}{l} \ell \| m \Rightarrow \angle 1 \cong \angle 2 \\ m \| n \Rightarrow \angle 3 \cong \angle 4 \\ \angle 2 \cong \angle 3 \end{array}\right\} \Rightarrow \angle 1 \cong \angle 4 \Rightarrow \ell \| n$$

$\boxed{}$

8. Prove that in Euclidean geometry, if $PQ = PR = PS$, and $Q - P - R$ then ΔQRS



is a right triangle.

proof: $QP = SP \Rightarrow \triangle QSP$ is isoceles $\Rightarrow$

$\angle Q \cong \angle 1$

$PS = PR \Rightarrow \triangle PSR$ is isoceles

$\Rightarrow \angle 2 \cong \angle R$

Now $m\angle Q + m\angle R + (m\angle 1 + m\angle 2) = 180$

$2m\angle 1 + 2m\angle 2 = 180 \Rightarrow m\angle 1 + m\angle 2 = 90$

$\Rightarrow$ ~~$\angle QSR =$~~

$\triangle QSR$ is a right triangle.