

Name(Print) _____ Section _____ Grade _____

Attention: Answers without supporting work shown on the paper will receive NO credits.

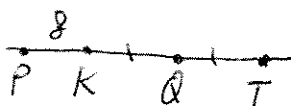
1. (3 points) The square of every real number x is nonnegative. Find the if-then conditional of this statements, and find its converse and contrapositive.

if $x \in \mathbb{R}$, then $x^2 \geq 0$

Converse: if $x^2 \geq 0$, then $x \in \mathbb{R}$.

Contrapositive: if $x^2 < 0$, then $x \notin \mathbb{R}$.

2. Given that K, P, Q , and T are points on a line such that $P-K-Q$ and $P-K-T$, Q is the midpoint of \overline{KT} , $PK = 8$, and $PT = 16$, find QT .



$$KT = PT - PK = 16 - 8 = 8$$

$$QT = \frac{1}{2} KT = 4$$

5

3. Consider the following relationships among three angles: $\angle 1$ is supplementary to $\angle 2$ and complementary to $\angle 3$, and $\angle 2$ and $\angle 3$ are a linear pair. Find $m\angle 1$.

$$m\angle 1 + m\angle 2 = 180 \text{ (1)}$$

$$m\angle 1 + m\angle 3 = 90 \text{ (2)}$$

$$m\angle 2 + m\angle 3 = 180 \text{ (3)}$$

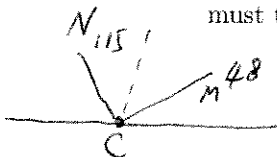
$$\text{(1)} - \text{(2)}: m\angle 2 - m\angle 3 = 90 \text{ (4)}$$

$$\text{(4)} + \text{(3)}: 2m\angle 2 = 270 \Rightarrow m\angle 2 = 135^\circ$$

$$\Rightarrow m\angle 1 = 45$$

3

4. Under the Protractor Postulate, the coordinates of rays \overrightarrow{CM} and \overrightarrow{CN} , are, respectively, 48 and 115 with respect to some half-plane containing \overrightarrow{CM} and \overrightarrow{CN} . What must the coordinate of the bisector of $\angle MCN$ be?



$$\text{it is: } 48 + \frac{1}{2}(115 - 48)$$

$$= 48 + \frac{1}{2} \times 67 = 81.5$$

3

5. Let two lines ℓ , and m lie in the plane P , discuss the possible relations between them.

$$\text{(1)} \ell \parallel m$$

$$\text{(2)} \ell \text{ intersects } m \text{ at one unique point.}$$

$$\text{(3)} \ell = m \text{ (coincident)}$$

4

6. Prove that for $A \neq B$, $\overrightarrow{AB} \subseteq \overrightarrow{AB} \subseteq \overleftrightarrow{AB}$.



$$\begin{aligned}\overrightarrow{AB} &= \{X : A-X-B, X=A, \text{ or } X=B\} \\ \overrightarrow{AB} &= \{X : A-X-B, A-B-X, X=A, \text{ or } X=B\} \\ \overleftrightarrow{AB} &= \{X : X-A-B, A-X-B, A-B-X, X=A, \text{ or } X=B\}\end{aligned}$$

Then it is clear $\overrightarrow{AB} \subseteq \overrightarrow{AB} \subseteq \overleftrightarrow{AB}$

4

7. Prove that if A , B , and C are any three distinct, collinear points, then either $A-B-C$, $A-C-B$, or $B-A-C$.

Let a coordinate system be correspondent to line AB , such that $A[a]$, $B[b]$, $C[c]$ as guaranteed by the Ruler postulate, then we must have either $a < b < c$, or $a < c < b$, or $b < a < c$. Then each case corresponds to $A-B-C$, $A-C-B$, or $B-A-C$ respectively.

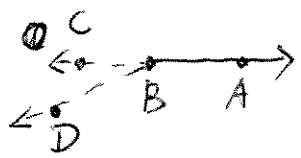
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8. Prove two angles that are complementary to the same angle have equal measures.

Let $m\angle 1 + m\angle 3 = 90$ and $m\angle 2 + m\angle 3 = 90$
then by simple algebraic operation, we must have $m\angle 1 = m\angle 2$.

4

9. Show that the opposing ray of a given ray is unique.



Let \overrightarrow{BC} be the opposing ray of \overrightarrow{BA} , by definition we have $C-B-A$. Assume \overrightarrow{BD} is another opposing ray of \overrightarrow{BA} , then again we obtain $D-B-A$. This shows C and D are on \overleftrightarrow{AB} , which implies $\overrightarrow{BC} = \overrightarrow{BD}$. \square

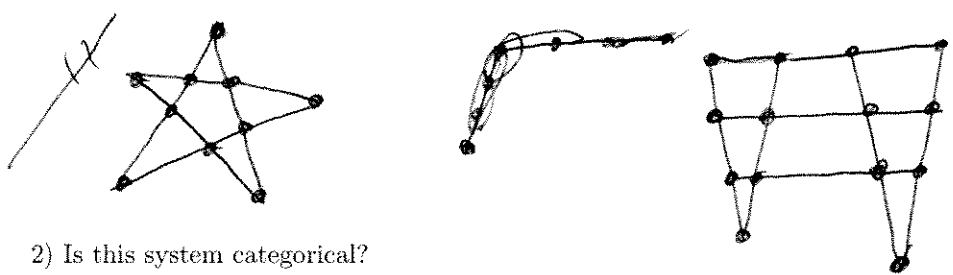
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10. Consider the following axiomatic system:

Undefined terms: point, line
AXIOMS:

1. Each line is a set of four points.
2. Each point is contained by precisely two lines.
3. Two distinct lines that intersect do so in exactly one point.

model



2) Is this system categorical?

NO