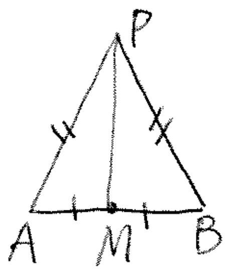
Math 372 Exam 2 Total pages: 2 Total points: 50 8
Instructor: Yi Wang

Name(Print)_______________ Section _______________ Grade__________

Attention: **Answers without supporting work shown on the paper will receive NO credits.**

1. (5 points) Prove the statement: If $PA = PB$ and M is the midpoint of segment $\overline{AB}$, then line $\overleftrightarrow{PM}$ is perpendicular to segment $\overline{AB}$.



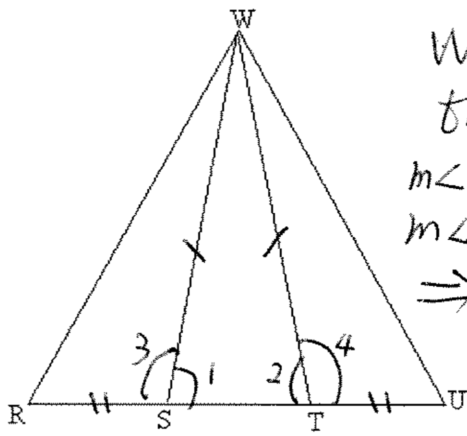
$\left.\begin{array}{l} PA=PB \\ AM=BM \\ PM=PM \end{array}\right\} \Rightarrow \triangle PAM \cong PBM$ by SSS

$\Rightarrow \angle AMP \cong \angle BMP$ by CPCF.

$m\angle AMP + m\angle BMP = 180$ since they are a linear pair.

Therefore $m\angle AMP = m\angle BMP = 90$, i.e. $\overline{PM} \perp \overline{AB}$ ▨

2. (5 points) If $WS = WT$ and $RS = TU$, with $R-S-T-U$, prove that $\angle R \cong \angle U$.



$WS = WT \Rightarrow \triangle WST$ is an isosceles triangle $\Rightarrow \angle 1 \cong \angle 2$

$m\angle 1 + m\angle 3 = 180$, since $\angle 1$ and $\angle 3$ are a linear pair.

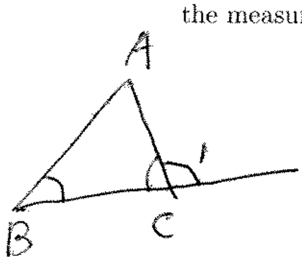
$m\angle 2 + m\angle 4 = 180$ since $\angle 2$ and $\angle 4$ are a linear pair.

$\Rightarrow \angle 3 \cong \angle 4$.

$\Rightarrow \triangle RWS \cong \triangle UWT$ by SAS

$\Rightarrow \angle R \cong \angle U$ by CPCF ▨

3. 1) Use the Exterior Angle Inequality to prove the following statement: the sum of the measures of any two angles of a triangle is less than 180.



Let $\triangle ABC$ be any triangle, without loss of generality, we prove $m\angle B + m\angle BCA < 180°$. Notice $m\angle BCA + m\angle 1 = 180$ because the two angles form a linear pair. Now $m\angle 1 > m\angle B$ by Exterior Angle Inequality, we then have

$m\angle BCA + m\angle B < 180°$ ▨

2) Use Saccheri-Legendre Theorem to prove that a triangle can have at most one right or obtuse angle.

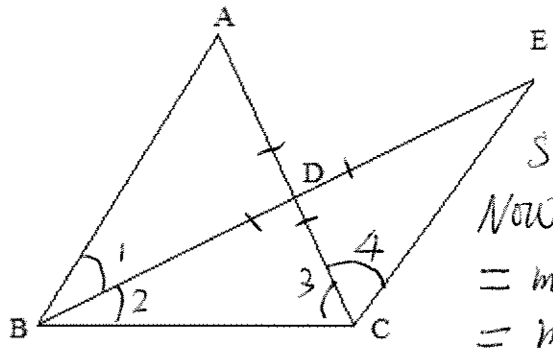
Saccheri-Legendre theorem: The angle sum of any triangle (in absolute geometry) can not exceed 180.

Let $\triangle ABC$ be given, Assume $\angle A$ and $\angle B$ are two right or obtuse angles without loss of generality, then

$$m\angle A + m\angle B + m\angle C \geq 180 + m\angle C > 180$$

which contradicts to Saccheri-Legendre THM. ▨

4. In the following figure, D is the midpoint of $\overline{AC}$ and $\overline{BE}$ as well. Show that the angle sum of ΔABC equals to the angle sum of ΔEBC.



We first note

$\triangle ABD \cong \triangle CED$ by SAS.

So $\angle 1 \cong \angle E$ and $\angle A \cong \angle 4$.

Now the angle sum of $\triangle ABC$

$= m\angle A + m\angle 1 + m\angle 2 + m\angle 3$

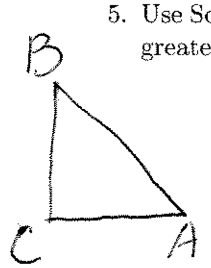
$= m\angle 4 + m\angle E + m\angle 2 + m\angle 3$

$= m\angle E + m\angle 2 + (m\angle 3 + m\angle 4)$

$=$ the angle sum of $\triangle EBC$

▨

5. Use Scalene Inequality to prove that the hypotenuse of a right triangle has measure greater than that of either leg.



Let $\triangle ABC$ be a right triangle with $\angle C$ being a right angle. Since a triangle can have at most one right angle or obtuse angle, $m\angle C > m\angle A$ and $m\angle C > m\angle B$. It then follows that $AB > AC$ and $AB > BC$. ▨

6. Use the triangle Inequality to prove that for any three points A, B and C,

$$AB - BC \leq AC \leq AB + BC.$$

1. by triangle inequality, we have $AC \leq AB + BC$

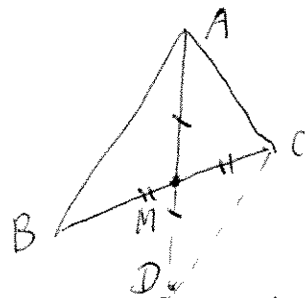
2. We also have $AB \leq AC + BC$, i.e. $AB - BC \leq AC$

3. Therefore $AB - BC \leq AC \leq AB + BC$. ▨

7. Suppose that $\overline{AM}$ is the median to side $\overline{BC}$ of ΔABC. Prove

$$AM < \frac{1}{2}(AB + AC)$$

using the Triangle Inequality.



~~Assume~~ extend $\overline{AM}$ to D such that $AM = MD$.

$\Rightarrow \Delta ABM \cong DCM$ by SAS

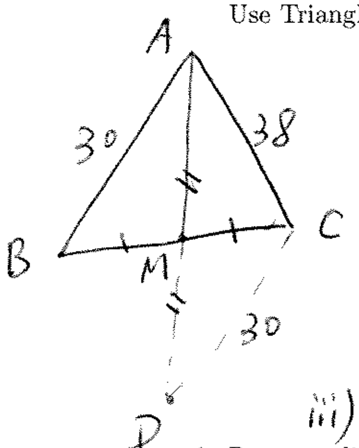
$\Rightarrow AB = CD$ by CPCF

In ΔADC, apply triangle inequality, $AD < AC + CD$

$\Rightarrow 2AM < AC + AB \Rightarrow AM < \frac{1}{2}(AB + AC)$ ☒.

8. Suppose that $\overline{AM}$ is the median to side $\overline{BC}$ of ΔABC. $AB = 30$ and $AC = 38$. Use Triangle Inequality to show that

$$4 < AM < 34.$$



i) From problem 7, We have $AM < \frac{1}{2}(AB + AC)$
$= \frac{1}{2}(30 + 38)$
$= 34$

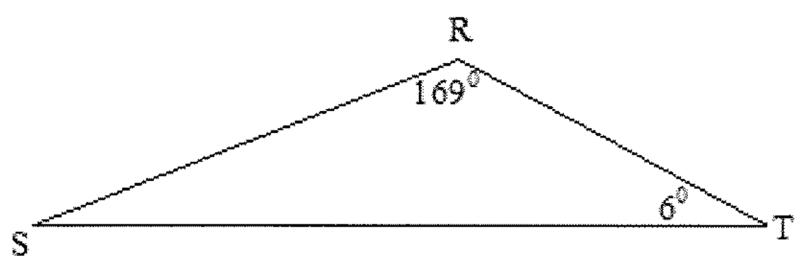
ii) extend $\overline{AM}$ to to D such that $AM = MD$. then $\Delta ABM \cong DCM \Rightarrow CD = AM$ by CPCF.

In ΔADC, ~~$AD = 2AM$~~ $AD + DC > AC$

i.e. $2AM + 30 > 38 \Rightarrow 2AM > 38 - 30 \Rightarrow AM > 4$.

iii) Therefore $4 < AM < 34$. ☒

9. Prove or disprove: in $\triangle RST$ with angle measures as indicated, $RS \geq RT$.



~~since~~

~~i) The sum of the measures of any two angles of a triangle~~ ~~is less than~~ ~~can not exceed 180°, so $m\angle S < 180° - 169° = 11°$~~

~~ii) So R We conclude $RS \geq RT$ is not necessarily correct~~

~~A counter example can be~~

~~ii) on the other hand~~ since the angle sum of any triangle $\leq 180°$.

so $m\angle S \leq 180° - 169° - 6° = 5°$

We definitely have $m\angle S < m\angle T$

$\Rightarrow RS \geq RT$ by Scalene Inequality. ☒